WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

ELSACOR02T-ELECTRONICS (CC2)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *five* questions 2×5 = 10

1. Define orthogonal matrix. Show that if A is an orthogonal matrix, then $|A| = \pm 1$.

2. Define Beta and Gamma function.

3. Define trace of a matrix.

4. Define Hermitian Matrix. Show that the matrix $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ is Hermitian matrix.

5. Prove that the adjoint of a diagonal matrix of order 3 is also diagonal.

6. Prove that: $\int_0^{\pi/2} \int_0^{\pi} \cos(x+y)\,dxdy = -2$

7. Solve: $(D^2+9)y = \sin 3x$

8. State Fuchs' theorem for the validity of series solution of differential equation.

GROUP-B

Answer any *six* questions 5×6 = 30

9. Solve the following differential equations: 2+3

   (a) $\frac{dy}{dx} + \ln x^y = 0$, (b) $2xy\frac{dy}{dx} = x^2 + y^2$

10. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ 5

11. Find the power series solution of the following differential equation: 5

    $$\frac{d^2y}{dt^2} + w^2 y = 0$$

12. Solve the system of equations by Gauss-elimination method, 5

$$2x_1 + 3x_2 + x_3 = 9$$
$$x_1 + 2x_2 + 3x_3 = 6$$
$$3x_1 + x_2 + 2x_3 = 8$$

13. Solve with the help of matrices, the simultaneous equations: 5

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

14.(a) Write down the Bessel's equation of order n. Starting from expression of Bessel's function of first kind of order n i.e., $J_n(x)$, prove the following recurrence formulae: 1+2+2

(b) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

(c) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

15. Find the matrix A^{-1} using Caley-Hamilton's theorem when 5

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

16. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Is the matrix A unitary? 4+1

17. Evaluate the following two integrals using gamma functions, $2\frac{1}{2} + 2\frac{1}{2}$

(a) $\int_0^1 \ln\left(\frac{1}{x}\right) dx$

(b) $\int_0^{\pi/2} (\tan^3\theta + \tan^5\theta) e^{-\tan^2\theta} \, d\theta$

18. Write down the Rodrigue's formula for Legendre polynomial $P_n(x)$ and Prove that: 1+4

$$P_n'(x) = x P_{n-1}'(x) + n P_{n-1}(x)$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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