WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Why do we use minimum ratio criterion in Simplex method?

(b) Prove that the transportation problem always has a feasible solution.

(c) Find the number of basic feasible solution of the following LPP:

$$\text{Maximize} \quad z = 2x_1 + 3x_2$$
$$\text{Subject to} \quad x_1 + 2x_2 \geq 1$$
$$x_1 - x_2 \geq 1$$
$$x_1,\ x_2 \geq 0$$

(d) What is the criterion for no feasible solution in two-phase method?

(e) Prove that if the primal problem has an unbounded objective function, then the dual has no feasible solution.

(f) If the stock in each origin is one unit and requirements of every supplier is one unit in a square transportation problem of order *m* then how many basic variables will be zero in initial basic feasible solution.

(g) Is (2, 1) a feasible solution of the following LPP?

$$\text{Maximize} \quad z = x_1 - 2x_2$$
$$\text{Subject to} \quad x_1 + x_2 \geq 2$$
$$x_1 - x_2 \leq 1$$
$$x_1,\ x_2 \geq 0$$

(h) If the pay-off matrix is skew-symmetric of a two-person zero-sum game then what will be the game value.

(i) Find the extreme points of the convex set $\{(x_1, x_2) : x_1 + x_2 \leq 1,\ x_1 - x_2 \leq 3\}$.

(j) Examine whether the set $X = \{(x_1, x_2) : x_1 x_2 \leq 4\}$ is convex.

2. (a) A business manager has the option of investing money in two plans. Plan *A* guarantees that each rupee invested will earn 70 paise a year and plan *B* guarantees that each rupee invested will earn Rs. 2.00 every two years. In plan *B*, only investments for periods that are multiples of 2 years are allowed. How should the manager invest Rs. 50,000/- to maximize the earnings at the end of 3 years? Formulate the problem as a Linear Programming Problem. 4

(b) Solve the following L.P.P. using graphical method 4

Maximize $z = 6x_1 + 10x_2$

Subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$

3. (a) If the feasible region of a linear programming problem is strictly bounded and contains a finite number of extreme points then prove that the objective function of the linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solutions. 4

(b) $x_1 = 1, x_2 = 1, x_3 = 2$ is a feasible solution of the system of equations $2x_1 + x_2 + x_3 = 5$, $x_1 + 3x_2 + x_3 = 6$. Reduce the feasible solution to two different basic feasible solutions. 4

4. (a) Solve the following L.P.P. by two phase method 6

Minimize $z = 4x_1 + x_2$

Subject to $x_1 + 2x_2 \leq 3$

$4x_1 + 3x_2 \geq 6$

$3x_1 + x_2 = 3$, and $x_1, x_2 \geq 0$

(b) Prove that $S = \{(x, y) \in E^2 : |x| \leq 2, |y| \leq 1\}$ is a convex set. 2

5. Use Charne's penalty method to 8

Maximize $Z = x_1 + 2x_2 + x_3$

Subject to $x_1 + 0x_2 + 2x_3 \leq 5$

$2x_1 + x_2 + 0x_3 \leq 4$

$x_1 + x_2 + x_3 \geq 1$

$x_1, x_2, x_3 \geq 0$

6. (a) State Fundamental theorem of duality. 2

(b) Solve the dual of the following L.P.P (primal) then obtain the solution of the primal 6

Maximize $Z = x_1 + x_2$

Subject to $x_1 \leq 4$

$x_2 \leq 2$

$3x_1 + 2x_2 \leq 12$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

7. (a) Find the optimal assignment from the following profit matrix: 5

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

(b) If a constant be added to any row and / or any column of the cost matrix of an assignment problem, then prove that the resulting assignment problem has the same optimal solution as the original problem. 3

8. (a) Solve the following transportation problem: 6

	D_1	D_2	D_3	D_4	a_i
O_1	10	20	5	7	10
O_2	11	9	12	8	20
O_3	4	16	7	9	30
O_4	14	7	1	0	40
O_5	3	12	5	19	50
b_j	60	60	20	10	

(b) Prove that the number of basic variables in a transportation problem is at most $(m+n-1)$. 2

9. (a) Find the optimal assignment to find the minimal cost for the assignment problem with the following cost matrix: 6

	I	*II*	*III*	*IV*	*V*
A	8	2	×	5	4
B	10	9	2	8	4
C	5	4	9	6	×
D	3	6	2	8	7
E	5	6	10	4	3

(b) Find the initial B.F.S. of the following transportation problem by matrix minima method: 2

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
b_j	16	18	31	25	

10.(a) Prove that the set of optimal strategies for each player in $m \times n$ matrix game is a convex set. 4

(b) Solve the game problem by reducing it into 2×2 problem with the help of dominance property. 4

$$\begin{pmatrix} 7 & -4 & 3 & -3 & -4 \\ 5 & 4 & 2 & 4 & 5 \\ 4 & 5 & 3 & -1 & 2 \\ 6 & 7 & 3 & -2 & -3 \end{pmatrix}$$

11.(a) In a two person zero sum game for what condition, the 2×2 payoff matrix will have no saddle point. 2

(b) Solve graphically the following game problem: 6

	B_1	B_2	B_3	B_4
A_1	1	2	4	7
A_2	7	4	2	1

12. Solve the following game problem by converting it into a L.P.P.: 8

		Player B		
		B_1	B_2	B_3
Player A	A_1	1	1	–1
	A_2	1	–1	1
	A_3	–1	1	1

13.(a) If s^{th} row of the payoff matrix of an $m \times n$ rectangular game be dominated by its r^{th} row of the payoff matrix, then prove that the deletion of s^{th} row from the pay-off matrix does not change the set of optimal strategies of the row player (maximizing player). 6

(b) Find the optimal strategies and game value of the following game whose payoff matrix is given by: 2

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	19	5	2	14
	A_2	10	4	6	13
	A_3	12	15	8	5
	A_4	17	14	9	12
	A_5	5	13	7	17

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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