WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

PHSADSE02T-PHYSICS (DSE1/2)

ADVANCED DYNAMICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
Answer must be precise and to the point to earn credit.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***fifteen*** questions from the following: 2×15 =30

(a) What is a semi-holonomic constraint? Give an example.

(b) What do you mean by forces of constraint in a system?

(c) A bead is sliding along a smooth massless circular ring that is rotating about its diameter. State the nature of constraints present in the system.

(d) What is the significance of fundamental Poisson bracket in connection with the canonical transformations?

(e) If all the generalised coordinates $(q_1, q_2, ..., q_n)$ of a system become cyclic what will be the time dependence of those coordinates?

(f) What is the value of x if the following transformation is canonical?

$$Q = \sqrt{2q}e^{-1-2x}\cos(p),\ P = \sqrt{2q}e^{-1-x}\sin(p).$$

(g) State, with reasons, the number of degrees of freedom of a rigid body.

(h) Show that the angular velocity and the angular momentum of a rotating rigid body may not be parallel in general.

(i) A semi-circular arc and a quadrant of a circle, each of mass M and radius R, are rotating separately about an axis passing through the centres of their respective circles and perpendicular to their planes. Find the ratio of their moments of inertia.

(j) What is meant by a spherical top? Write down Euler's dynamical equations for it.

(k) Explain the difference between "local" and "global" minima in case of a generic potential for one dimensional motion.

(l) A particle of mass m in one dimensional motion along x-axis $(x > 0)$ with its potential energy given by $V(x) = x + 1/x$, is executing small oscillation near its stable equilibrium. Find the frequency of the oscillation.

(m) Two identical point masses (each of mass m), connected by a massless spring (with spring constant k), are placed on a smooth horizontal table. Find the frequency of oscillation when the spring is stretched and then released.

(n) What is meant by a nonlinear dynamical system?

(o) Draw the 2D phase space diagram of a point particle of mass *m* falling freely under the action of earth's gravity.

(p) Show that an oscillatory motion of a system far away from its stable equilibrium configuration under an arbitrary potential, is nonlinear in general.

(q) Show that for a first order dynamical system (with, $\dot{x} = f(x)$), there are no periodic solutions.

(r) What is meant by a limit cycle in a dynamical system?

(s) Draw a typical phase space trajectory of damped harmonic oscillator under over-damped condition. What is the nature of its fixed point?

(t) Explain the idea of 'negative damping' in connection with van-der-Pol oscillator.

2. (a) Consider the planar motion of a point particle of mass *m* suspended from a point with the help of a massless rigid rod of length *l*. Find the expression of the force of constraint. 4+4+2

(b) Identify the fixed points on the phase space of the system and comment on their nature with justification.

(c) Find the frequency of small amplitude oscillation of a system. Will the frequency differ if the amplitude increases?

3. (a) Show that the Poisson bracket is invariant under canonical transformation. 4+4+2

(b) Four particles each of mass *m* are placed at the corners of a square of side *a* in the $x-y$ coordinate system. One corner of the square is coincident with the origin and its two sides lie along *x*-axis and *y*-axis respectively. $x'-y'$ coordinate system is obtained by rotating the *x*, *y* axes through an angle θ about an axis passing through the origin and perpendicular to the plane of the square. What is the value of θ if the x', y' axes be the principal axes for the system of masses?

(c) Determine the points of stable and unstable equilibrium for a potential

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{3}x^3,$$

and sketch the typical nature of phase trajectories near those points.

4. (a) A light string is stretched with tension *F* between two fixed points A and D on a frictionless horizontal table. Two point particles B and C, each of mass *m*, are attached to the string at the points of trisection. Consider the small oscillations of the particles in the plane of the table and at right angle to the string. (1+2+2+1) + 2+2

(i) Draw a neat, labeled diagram of the arrangement.

(ii) Derive the equations of motion for the above system.

(iii) From the eigenvalue equation find the positive roots of the system.

(iv) Draw two diagrams to illustrate the normal modes of oscillation of the system. Clearly label the phase and the anti-phase oscillations.

(b) Find all the fixed points for $\dot{x} = x^2 - 1$, and classify them according to their nature of stability.

(c) A bead (of mass *m*) slides on a uniformly rotating (with angular velocity ω) straight wire in a force-free space. Find its Lagrangian.

5. (a) Find the canonical transformation generated by (2+1+1+2) +4

$$F_1(Q, q) = \lambda q^2 \cot Q,$$

λ being a constant. If the Hamiltonian in (q, p) representation is

$$H(q, p) = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2,$$

find the Hamiltonian in (Q, P) representation. Choose λ to make this Hamiltonian independent of Q and hence find the equation of motion in each representation.

(b) Hamiltonian of an anharmonic oscillator is given by,

$$H(q, p) = \frac{p^2}{2m} + x^4 - x^2 + \alpha^2 x^2,$$

α being a positive parameter. Find the value of α across which a bifurcation occurs in the system, clearly showing the change of the number and nature of the fixed points in the system across that value of α.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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