

2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTMC402X1

[Classical Mechanics]

Full Marks : 25

Time : One Hour

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.
Notation and symbols have their usual meanings.*

Group - A

Answer any *two* questions : $2 \times 2 = 4$

1. Find the expression for the generalised force in terms of generalised coordinates. [CO1]
2. If $Q = ap + bq$, $P = cp + dq$ is a canonical transformation, then find the relation among a, b, c, d . [CO2]
3. "The Poisson bracket of two constants of motion is itself a constant of motion." — Explain it. [CO2]
4. Set up the Hamiltonian for a system whose Lagrangian is given by $L(x, \dot{x}) = \frac{1}{2}x^2 - \frac{1}{2}w^2\dot{x}^2 - \alpha x^3 + \beta x\dot{x}^2$,
where α, β and w are constants. [CO1]

P.T.O.

(2)

Group - B

Answer any *two* questions : 4×2=8

5. State and prove the Poincaré theorem. [CO2]
6. If the equations of transformation do not depend explicitly on time and the potential energy is velocity independent, then prove that H is the total energy of the system. [CO1]

7. Prove that $J = \int_{x_0}^{x_1} F(y, y', x) dx$

will be stationary only if $\frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) - \frac{\partial F}{\partial y} = 0$. [CO1]

8. Let $G = G_1(q_1, q_2, \dots, q_n, Q_1, Q_2, \dots, Q_n, t)$ be a generating function of a canonical transformation. Prove that $p_j = \frac{\partial G_1}{\partial q_j}$, $P_j = -\frac{\partial G_1}{\partial Q_j}$ for all j . Hence, prove that if the canonical transformation is given, then one can determine the generating function. [CO2]

Group - C

Answer any *one* question : 8×1=8

9. Suppose a particle of mass m_0 is moving with a velocity v , then show that its mass at any time is given by

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}, \text{ where } c \text{ is the speed of light. [CO2]}$$

(3)

10. State the principle of least action. Deduce Hamilton's equations of motion from this principle. [CO1]

Internal Assessment : 5 marks
