

2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTMC402X2

[Abstract Algebra]

Full Marks : 25

Time : One Hour

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Notations and symbols have their usual meanings.

Group - A

Answer any two questions : $2 \times 2 = 4$

1. Give an example of a Principal ideal domain which is not Euclidean domain. [CO1]
2. Let $f : G \rightarrow H$ be a homomorphism of groups. Show that $f(G') \subseteq H'$. [CO1]
3. Show with example that finite extensions need not be simple. [CO2]
4. Let D be a Euclidean domain with a Euclidean valuation v . If $a \mid b$ and $v(a) = v(b)$, then a and b are associates in D . [CO1]

P.T.O.

(2)

Group - B

Answer any *two* questions : $4 \times 2 = 8$

5. Show that the group A_n is generated by 3-cycles. Also show that all 3-cycles are conjugates in A_n if $n \geq 5$.

[CO2]

6. Prove or disprove that the domain $D = \mathbb{Z}[\sqrt{-5}]$ is a UFD.

[CO1]

7. If G is solvable and $H \triangleleft G$, show that G/H is solvable.

[CO2]

8. If K is algebraic over F and L is algebraic over K , then show that L is algebraic over F .

[CO2]

Group - C

Answer any *one* question : $8 \times 1 = 8$

9. (i) Show that if R is a principal ideal domain, then it is a unique factorization domain. Give an example to show that the converse is not true. [CO1]
- (ii) Prove that subgroups of a solvable group are solvable. $6+2$ [CO2]
10. (i) Is the regular pentagon constructible by using ruler and compass only? [CO1]
- (ii) Prove that $\sqrt[3]{2}$ is not ruler and compass constructible. [CO2]

(3)

- (iii) Find a gcd of the elements $1 + 3i$ and $3 + 4i$ in $\mathbb{Z}[i]$. 4+2+2 [CO1]

Internal Assessment : 5 marks
