2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTME404A0 / B1 & B2

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.
Notations and symbols have their usual meanings.

Paper : MTME404A0

[Ordinary Differential Equation and Special Functions]

Group - A

Answer any ***four*** questions : 2×4=8

1. Prove $F(a,b,c;z)=(1-z)^{-a}F\left(a,c-b,c;\frac{z}{z-1}\right)$. [CO4]

2. Let $P_n(z)$ be the Legendre polynomial of dgree n such that $P_n(1)=1, n=1,2,3,...$ Find the value of $\int_{-1}^{1}(P_3(z))^2dz$. [CO4]

3. Define self-adjoint differential equation with an example. [CO1]

4. For what value of n the general solution of Bessel's differential equation will be of the form

$$W = AJ_n(z) + BJ_{-n}(z).$$ [CO4]

5. Green's function of the differential operator L of the non-homogeneous differential equation : $Lu(x) = f(x)$. [CO2]

6. What is the role of the weight function in a Sturm-Liouville problem? [CO1]

Group - B

Answer any *four* questions : 4×4=16

7. Prove that, $P_{2m}(0) = (-1)^m \dfrac{(2m)!}{2^{2m}(m!)^2}$. [CO4]

8. Prove that the zeros of Legendre polynomial are all real, distinct and lie between -1 and 1. [CO4]

9. Express $z^4 + 2z^3 + 2z^2 - z - 3$ in terms of Legendre polynomial. [CO4]

10. Prove that, all the eigenvalues of a regular Strum-Liouville system with $r(x) > 0$, are real. [CO1]

11. Show that $J_0^2(z) + 2\sum_{n=1}^{\infty} J_n^2(z) = 1$ and prove that for real z, $|J_0(z)| < 1$, and $|J_n(z)| < \dfrac{1}{\sqrt{2}}$, for all $n \geq 1$. [CO4]

12. If the vector functions $\varphi_1, \varphi_2, ..., \varphi_n$ defined as follows:

$$\varphi_1=\begin{bmatrix}\varphi_{11}\\ \varphi_{21}\\ \cdot\\ \varphi_{n1}\end{bmatrix}, \varphi_2=\begin{bmatrix}\varphi_{12}\\ \varphi_{22}\\ \cdot\\ \varphi_{n2}\end{bmatrix}, ...\varphi_n=\begin{bmatrix}\varphi_{1n}\\ \varphi_{2n}\\ \cdot\\ \varphi_{nn}\end{bmatrix}$$

be n solutions of the homogeneous linear differential equation $\frac{dx}{dt} = A(t)x(t)$ in the interval $a \le t \le b$, then these n solutions are linearly dependent in $a \le t \le b$ iff Wronskian $W[\varphi_1, \varphi_2, ..., \varphi_n] = 0 \forall t$, on $a \le t \le b$. [CO3]

Group - C

Answer any *two* questions : 8×2=16

13. (i) Prove that if $f(z)$ is continuous and has continuous derivatives in [−1, 1] then $f(z)$ has unique Legendre series expansion is given by $f(z) = \sum_{n=0}^{\infty} C_n P_n(z)$ where P_n's are Legendre polynomials $C_{n=} \frac{2n+1}{2}\int_{-1}^{1} f(z)P_n(z)$, $n = 1,2,3...$ [CO4]

(ii) Express $J_4(z)$ in terms of $J_0(z)$ and $J_1(z)$.

5+3 [CO4]

P.T.O.

14. (i) Using Green's function method, solve the following differential equation $\frac{d^2y}{dx^2} - y = -2e^x$ with boundary conditions, $y(0) = y'(1)$, $y(l) + y'(l) = 0$. [CO2]

(ii) Prove that $J_n(z) = 0$ has no repeated roots except at $z = 0$, where $J_n(z)$ is the Bessel's function. 5+3 [CO4]

15. (i) Find the characteristics values and characteristic functions of the Sturm-Liouville problem $(xy')' + \frac{\lambda y}{x} = 0$; $y'(1) = 0$, $y(b) = 0$, $b > 1$. [CO1]

(ii) Let, $P_n(z)$ be the Legendre polynomial of degree $n \geq 0$. If $1 + z^{10} = \sum_{n=0}^{10} C_n P_n(z)$, then find the value of C_5. 5+3 [CO4]

16. (i) Deduce the integral formula for confluent hypergeometric function. [CO4]

(ii) Find the general solution of the non-homogeneous system,

$$\frac{dX}{dt} = \begin{pmatrix} 7 & 4 & 4 \\ -6 & -4 & -7 \\ -2 & -1 & 2 \end{pmatrix} X + \begin{pmatrix} 2t+3 \\ -3 \\ 3t-9 \end{pmatrix}, \text{ where } X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

2+6 [CO5]

Internal Assessment : 10 marks

Paper : MTME404B1/B2

(Data Science and Graph Theory)

MTME404B1

(Unit-I : Data Science)

Group - A

Answer any *two* questions : $2\times2=4$

1. What is meant by Euclidean Norm? Write its mathematical expression. Explain with example. [CO1]

2. What is topology of data? [CO1]

3. How spread of a data set can be measured? Explain with examples. [CO2]

4. What is a Box Plot? Mention any two components of a box plot. [CO2]

Group - B

Answer any *two* questions : $4\times2=8$

5. Suppose, you have a list of marks obtained in Mathematics, Physics, Chemistry and Biology for one hundred students in a school. Write a Python program to find the average result considering (a) all subjects, (b) any subject for all data and some missing data using two dimensional NumPy array. [CO2]

P.T.O.

6. Suppose, a fictional data set that describes the weather conditions for playing a game of football, is as follows :

Day	Outlook	Temperature	Humidity	Windy	Play Football
1	Rainy	Hot	High	False	Yes
2	Rainy	Hot	High	True	No
3	Overcast	Hot	High	False	Yes
4	Sunny	Cool	Normal	False	Yes
5	Sunny	Mild	High	True	No

Predict if football will be played based on the weather conditions (Outlook, Temperature, Humidity, Wind) as (Sunny, Hot, Normal, False) using Bayes' theorem /KNN. [CO1]

7. Find the first, second and third quartile of the following data set using Python.
3, 5, 7, 8, 10, 11, 3, 1, 1, 11, 15, 8, 10, 20, 7, 9, 12, 18, 14 [CO2]

8. Explain the Data Science Life Cycle with suitable steps. [CO1]

Group - C

Answer any *one* question : $8 \times 1 = 8$

9. What is the condition number of a matrix? Derive it using vector norm. How is it used to find whether a system of linear equations is stable or not? Write a Python program to find the condition number of a matrix.
1+2+2+3 [CO2]

10. What is the concept of dimension reduction? Mention different types of dimension reduction methods. Explain any one from these. 2+1+5 [CO1]

Paper : MTME404B2

(Unit-II : Graph Theory)

Group - A

Answer any *two* questions : 2×2=4

1. Define the term 'Hamiltonian circuit'. Also, draw a graph with six vertices containing an Eulerian circuit but not Hamiltonian circuit. [CO1]

2. Draw the undirected graph G corresponding to adjacency matrix A :

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ [CO2]

3. Define the term 'Binary tree' and write down its applications. [CO1]

4. Define cut set. How is it different from a cut vertex? 1+1 [CO1]

P.T.O.

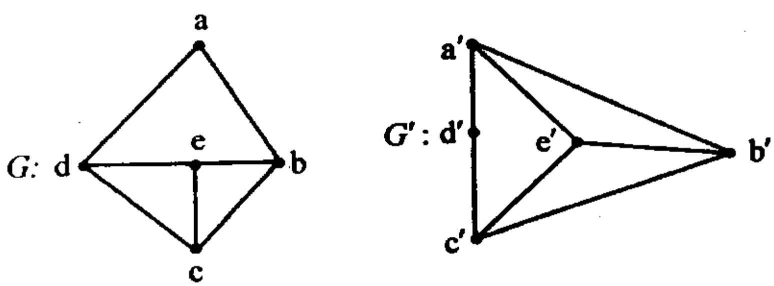
Group - B

Answer any *two* questions : 4×2=8

5. If G is a simple connected planar graph with $n(\geq 3)$ vertices and e edges, then prove that $e \leq 3n-6$ and using this, verify that the complete graph, K_5 is planar or not. [CO1]

6. In a connected graph G, prove that any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set. [CO2]

7. Show that the graphs G and G' are isomorphic :



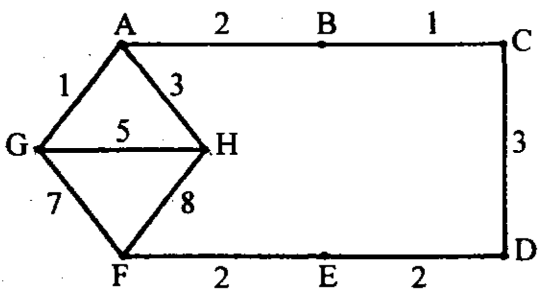
[CO1]

8. Define the term "Chromatic number" for the graph colouring with example. Find the chromatic number of the Wheel graph W_6. [CO2]

Group - C

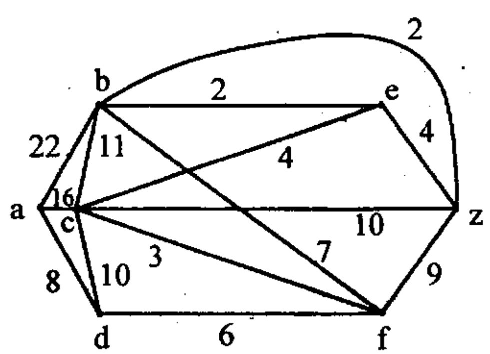
Answer any *one* question : $8\times1=8$

9. (i) Solve the travelling salesman problem for the following weighted graph :



[CO2]

(ii) Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to z :



3+5 [CO2]

P.T.O.

10. (i) Prove that the relation $\chi(G) \leq \Delta(G)+1$ for any graph G where $\chi(G)$ is the chromatic number and $\Delta(G)$ is the maximum degree of a vertex in G. [CO2]

(ii) Show that the chromatic number of a cycle with n vertices (C_n) is 2 if n is even and 3 if n is odd. [CO2]

(iii) Write down Chromatic polynomial of a 'tree' and 'complete graph' with n vertices. 3+3+2 [CO2]

Internal Assessment : 10 marks